

Social innovation as the community provision of public goods

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Abstract. In the last decades, social innovation has been widely discussed in academia, especially in sociology and political science, although a shared definition is still missing. To a large extent, academic economists were not part of the debate, which instead has flourished in policy circles. The aim of this paper is to develop a model to analyze social innovation as the *community* provision of a public good. On the “demand” side, we interpret social innovation as an imperfect substitute for a “traditional” group-specific public good, produced by the local government and financed by taxes. On the “supply side”, social innovation is co-produced by “suppliers” (social innovators) and “consumers” (the members of the target group, i.e. those citizens who take advantage of the local public good) with the former being motivated by altruism (or empathy) in favor of the latter. Our results focus on public policy towards social innovation. From a welfare point of view, traditional public good and social innovation should coexist, with governments also involved in actively supporting social innovation via subsidies. We also show that each community member can be better off with social innovation, as relying on social innovators’ intrinsic motivations can lower taxes used to finance traditional public goods and social innovation subsidies. Finally, we show that the optimal policy towards social innovation typically depends on the social innovation co-production function characteristics.

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1. Introduction

In the last decades, social innovation has been widely discussed in academia, especially in sociology and political science, although a shared definition is still missing (Mulgan, 2006; Godin, 2012; Moulaert and MacCallum, 2019; Manzini, 2015; Avelino et al., 2019, Cuntz et al., 2020). To a large extent, academic economists were not part of the debate (Rehfeld and Terstriep, 2020), which instead has flourished in policy circles (European Commission, 2013, 2017; Mulgan, 2019; Nicholls, A., & Edmiston, D., 2018).

The aim of this paper is to develop a model to analyze social innovation as the *community* provision of a group-specific, local public good. While the literature on the private provision of public goods is extremely rich (e.g. Bergstrom, Blume and Varian, 1986; Heal, 2021; Heal, 2022), our focus on communities and interaction between providers and users of the public good, in line with insights put forth by the literature on open innovation (Giordani et al, 2018), puts the social dimension at the core of the analysis.

On the “demand” side, we interpret social innovation as an imperfect substitute for a “traditional” group-specific public good, produced by the local government and financed by taxes. On the “supply side”, social innovation is co-produced by “suppliers” (social innovators) and “consumers” (the members of the target group, i.e. those citizens who take advantage of the local public good) with the former being motivated by altruism (or empathy à la Heal, 2021) in favor of the latter. Therefore, the social aspect in our view of social innovation appears both in individual preferences and in production function of social innovation, consistently with the extant literature (Ashraf and Bandiera, 2017; Bisin and Verdier, 2011; Andreoni, 2006; Besley and Ghatak, 2005; Besley and Ghatak, 2017).

The rest of the paper is organized as follows. In Section 2 we introduce the general version of the model, and we derive and compare the social optimum and the Nash equilibrium of the policy game between the local government and the community. In Section 3 we move to the analysis of two special cases, and obtain additional results. Section 4 provides a discussion of results in line of the extant literature and policy debate, while extending at the same time the model in a number of directions. Finally, Section 5 concludes.

2. The provision of social innovation: a general model

2.1 Description

2.1.1 The community structure

Our model is inspired by the literature in local public finance, considering the role of heterogeneity of citizens for public good provision (Tiebout, 1956; Bewley, 1981; Rubinfeld 1987; Alesina et al., 1999).

In our model we consider a community in a political jurisdiction composed by three different groups. The first group, denoted with T , is the target group, whose size is N_T . The members of this group are the actual recipients of a group-specific, local public good g . g can be produced in two ways. The first one is a “traditional” way, in which the local government produces g_P by collecting taxes. For the traditional public good, we assume that the production function is characterized by constant returns to scale, in that unit of resources invested translate in one unit of g_P . The second technology to produce g is through a process of social innovation, in which a public good g_I is produced through the direct contribution by community members, in the form of resources such as money and time. Preferences of T members are represented by:

$$U_T(g_P, g_I, x_T) = u_T(g_P, g_I) + x_T \quad (1)$$

where $u_T(g_P, g_I)$ represents the preferences of T members over the two variants of the public good and x_T is the amount of the numeraire good consumed by T members. Defining $u'_{TP} \equiv \frac{\partial u_T(g_P, g_I)}{\partial g_P}$, $u'_{TI} \equiv \frac{\partial u_T(g_P, g_I)}{\partial g_I}$, $u''_{TP} \equiv \frac{\partial^2 u_T(g_P, g_I)}{(\partial g_P)^2}$, $u''_{TI} \equiv \frac{\partial^2 u_T(g_P, g_I)}{(\partial g_I)^2}$ and $u''_{TPI} \equiv \frac{\partial^2 u_T(g_P, g_I)}{\partial g_P \partial g_I}$, we shall assume that $u'_{TP} \geq 0$, $u'_{TI} \geq 0$, $u''_{TP} \leq 0$ and $u''_{TI} \leq 0$ (i.e. $u_T(\cdot)$ is an increasing and concave function its arguments) and that $u''_{TPI} \leq 0$, i.e. the two types of public good are substitutes.

T members are endowed with an exogenous income Y_T . In the logic of co-production that is proper of social innovation, we shall assume that T members can allocate their income between the numeraire good and the contribution to g_I . We denote as g_T the total contribution of group T , which is equally shared among its members, so that $\frac{g_T}{N_T}$ is the individual contribution. We shall assume that T members do not pay income taxes, but can receive a subsidy λ_T per unit of contribution to social innovation. It follows that T members have a budget constraint:

$$Y_T = x_T + \frac{g_T}{N_T} (1 - \lambda_T) \quad (2)$$

The second group, denoted with S , is the group of *social innovators*. Its size is N_S . S members are those who can contribute to production of social innovation, together with T members. We denote

as g_S the total contribution by group S , which is equally shared among its members, so that $\frac{g_S}{N_S}$ is the individual contribution. We shall assume that:

$$g_I = G(g_S, g_T) \quad (3)$$

The properties of the “social production function” $G(\cdot)$ are key for our results, and for this reason are described and commented below. Although they do not directly consume g_I , also social innovators may benefit from g_P and g_I for two reasons. First, they may be motivated by altruism in favor of the members of the target group. Second, as for social innovation is concerned, they may be motivated by the “consumption” of social innovation as a relational good.¹

Preferences of S members are thus represented by:

$$U_S(g_P, g_I, x_S) = u_S(g_P, g_I) + x_S \quad (4)$$

where $u_S(g_P, g_I)$ represents the preferences of S members over the two variants of the public good and x_S is the amount of the numeraire good consumed by S members. Defining $u'_{SP} \equiv \frac{\partial u_S(g_P, g_I)}{\partial g_P}$, $u'_{SI} \equiv \frac{\partial u_S(g_P, g_I)}{\partial g_I}$, $u''_{SP} \equiv \frac{\partial^2 u_S(g_P, g_I)}{(\partial g_P)^2}$, $u''_{SI} \equiv \frac{\partial^2 u_S(g_P, g_I)}{(\partial g_I)^2}$ and $u''_{SPI} \equiv \frac{\partial^2 u_S(g_P, g_I)}{\partial g_P \partial g_I}$, we shall assume that $u'_{SP} \geq 0$, $u'_{SI} \geq 0$, $u''_{SP} \leq 0$ and $u''_{SI} \leq 0$ (i.e. $u_S(\cdot)$ is an increasing and concave function its arguments) and that $u''_{SPI} \leq 0$, i.e. the two types of public good are substitutes.

S members have an exogenous income Y_S , and they finance the traditional public good through a lump sum tax τ collected by the local government. At the same time, λ_S is a unit subsidy to the contribution to social innovation by S , transferred by the local government. It follows that the individual budget constraint is given by:

$$Y_S - \tau = x_S + \frac{g_S}{N_S} (1 - \lambda_S) \quad (5)$$

Finally, the third group denoted with C , is the group of *citizens*. Its size is N_C . Members of this group do not (directly) contribute to social innovation, but they may be also motivated by altruism in favor

¹ Since preferences are defined over social innovation, the model cannot consider explicitly warm glow preferences resulting for group S from the pleasure of “donating” g_S in favour of T members. These could be included modifying the equation (5) below to reduce the marginal cost of g_S .

of T members, so they have preferences defined over g_P and g_I , and they finance the traditional public good by paying the lump sum tax τ from their income Y_C . It follows that their preferences are represented by:

$$U_C(g_P, g_I, x_C) = u_C(g_P, g_I) + x_C \quad (6)$$

where x_C is the amount of the numeraire good consumed by C members. As for social innovators, defining $u'_{CP} \equiv \frac{\partial u_C(g_P, g_I)}{\partial g_P}$, $u'_{CI} \equiv \frac{\partial u_C(g_P, g_I)}{\partial g_I}$, $u''_{SP} \equiv \frac{\partial^2 u_C(g_P, g_I)}{(\partial g_P)^2}$, $u''_{SI} \equiv \frac{\partial^2 u_C(g_P, g_I)}{(\partial g_I)^2}$ and $u''_{SPI} \equiv \frac{\partial^2 u_C(g_P, g_I)}{\partial g_P \partial g_I}$, we shall assume that $u'_{CP} \geq 0$, $u'_{CI} \geq 0$, $u''_{CP} \leq 0$ and $u''_{CI} \leq 0$ (i.e. $u_C(\cdot)$ is an increasing and concave function its arguments) and that $u''_{SPI} \leq 0$, i.e. the two types of public good are substitutes. As for the individual budget constraint, this is given by:

$$Y_C - \tau = x_C \quad (7)$$

2.1.2 The social innovation production function

We shall assume that $G(\cdot)$ is increasing and concave in its arguments: $G'_S \equiv \frac{\partial g_{ST}}{\partial g_S} > 0$, $G'_T \equiv \frac{\partial g_{ST}}{\partial g_T} > 0$, $G''_S \equiv \frac{\partial^2 g_{ST}}{(\partial g_S)^2} > 0$, $G''_T \equiv \frac{\partial^2 g_{ST}}{(\partial g_T)^2} < 0$. As for $G''_{ST} \equiv \frac{\partial^2 g_{ST}}{\partial g_S \partial g_T}$, we allow both $\frac{\partial^2 g_{ST}}{\partial g_S \partial g_T} > 0$ (i.e. the efforts of social innovators and target group members are complementary) and both $\frac{\partial^2 g_{ST}}{\partial g_S \partial g_T} < 0$ (i.e. the efforts of social innovators and target group members are substitutes). The first case applies when social innovators contribute to the social innovation project with specific skills and advanced human capital. This may be case for projects associated to health and (advanced) education, or projects to which social innovators bring significant managerial competences. The second case applies instead when the contributes by members in group T and S are relatively similar, as in the case in which the main input in the social innovation project is time “per se”, or monetary transfers.

In order to conduct the supermodularity analysis in Section 3 and the comparative statics exercise for Section 4 we shall assume that $G(\cdot)$ can be written as $g_I = \rho \bar{G}(g_S, g_T)$ so that variation in ρ can capture Hicks-neutral changes in the production function of social innovation, due for instance to technological or institutional innovations.

2.2 Results

2.2.1 The public good social optimum

As a benchmark, we determine the levels of g_P , g_S and g_T that maximize social welfare, defined as:

$$W(g_P, g_S, g_T) = N_T[u_T(g_P, g_I) + x_T] + N_S[u_S(g_P, g_I) + x_S] + N_C[u_C(g_P, g_I) + x_C] \quad (8)$$

i.e., the unweighted sum of all individuals' utility in the community. The constraints are given by the individuals budget sets (eqs (2), (5), (7)), by the social innovation production function (eq (3)) and by the government balanced budget constraint, i.e.:

$$g_P = \tau(N_S + N_C) - \lambda_S g_S - \lambda_T g_T \quad (9)$$

Plugging all the constraints into (8), and deriving with respect to g_P , g_S and g_T yields:

$$N_T u'_{TP}(g_P^*, g_I^*) + N_S u'_{SP}(g_P^*, g_I^*) + N_C u'_{CP}(g_P^*, g_I^*) = 1 \quad (10)$$

$$N_T u'_{TI}(g_P^*, g_I^*) G'_S(g_S^*, g_T^*) + N_S u'_{SI}(g_P^*, g_I^*) G'_S(g_S^*, g_T^*) + N_C u'_{CI}(g_P^*, g_I^*) G'_S(g_S^*, g_T^*) = 1 \quad (11)$$

$$N_T u'_{TI}(g_P^*, g_I^*) G'_T(g_S^*, g_T^*) + N_S u'_{SI}(g_P^*, g_I^*) G'_T(g_S^*, g_T^*) + N_C u'_{CI}(g_P^*, g_I^*) G'_T(g_S^*, g_T^*) = 1 \quad (12)$$

when superscript * denotes the social welfare maximizing levels. We observe that plugging (10) into (11) and (12) yields $G'_S(g_S^*, g_T^*) = 1$ and $G'_T(g_S^*, g_T^*) = 1$, which implies that socially optimum level of social innovation depends exclusively on the properties of social innovation production function.

2.2.2 The policy game between the government and the actors of social innovation

We now assume that the local government, group T and group S play a simultaneous game in which i) the local government chooses g_P in order to maximize social welfare (equation (8)) under the balanced budget constraint (9); ii) group T and group S choose g_T and g_S in order to maximize the individual utility of each group member (respectively, (1) and (4)) under his budget constraint (respectively (2) and (5)). Therefore, we treat each group as a single actor, ignoring any form of strategic interaction within each group.

The first order conditions which determine the Nash equilibrium of the game are:

$$N_T u'_{TP}(g_P^E, g_I^E) + N_S u'_{SP}(g_P^E, g_I^E) + N_C u'_{CP}(g_P^E, g_I^E) = 1 \quad (13)$$

$$N_S u'_{SI}(g_P^E, g_I^E) G'_S(g_S^E, g_T^E) = 1 - \lambda_S \quad (14)$$

$$N_T u'_{TI}(g_P^E, g_I^E) G'_T(g_S^E, g_T^E) = 1 - \lambda_T \quad (15)$$

where superscript E denotes the Nash equilibrium levels.

2.2.3 Comparing the social optimum and the Nash equilibrium in the policy game

The first result we derive involves the comparison between the social optimum and the Nash equilibrium when $\lambda_S = \lambda_T = 0$, i.e. when the government does not financially intervene in the provision of social innovation. The proof is in the Appendix.

Proposition 1 *If $\lambda_S = \lambda_T = 0$, $g_P^E \geq g_P^*$ and $g_I^E \leq g_I^*$.*

Proposition 1 shows that absent a financial government intervention subsidizing social innovation, the outcome of the policy game entails a level of the traditional public good that is higher than the social optimum, and a level of social innovation that is lower. As we show in Section 4, this does not necessarily mean that is not possible to guarantee T members the same level of utility they would get at the social optimum. However, the *composition* of the public good, in terms of traditional public good and social innovation, differ between the Nash equilibrium and the social optimum, and so it is socially inefficient. The intuition lies in the existence of the positive externality created both by g_T and g_S in favour of the other groups, which leads to a sub-optimal level when they are chosen to maximize T and S members utility. Although the government may overinvest in the traditional public good in order to compensate, it will do so by collecting taxes. Taxation and private contributions by T and S members are not equivalent forms of financing a public good, since they are associated to different implicit marginal costs for public good provision, and social innovation can rely in particular on the intrinsic motivation by social innovators.

An easy corollary of Proposition 1 is that subsidizing social innovation is a welfare improving policy. In Section 4 we derive the optimal social innovation policies for the two special cases.

2.2.4 The complementarity between the traditional public good and social innovation: a supermodularity analysis

How do equilibrium levels vary with model parameters? While standard comparative statistics exercises are performed on the special cases analyzed in Section 4, the general model is suitable for

supermodularity analysis (Amir, 2005), which analyze how the endogenous variables (g_p , g_T and g_S) co-move when exogenous parameters vary. In this section, we analyze the impact of all the six parameters: the three parameters directly related to social innovation production (ρ , λ_S and λ_T), and the three parameters related to the size of each group (N_T , N_S and N_C). What we do is to identify the condition for the game described in 2.2.2 to be supermodular in each of them. Since we assumed that $u''_{CPI} \leq 0$ and $u''_{SPI} \leq 0$ (so that the traditional public good and social innovation are substitutes in the preferences of the target group and social innovators) we first define $\hat{g}_p = -g_p$. In addition, we define $\hat{N}_C = -N_C$.

Proposition 2. Suppose $\frac{\bar{g}_{ST}''}{\bar{g}_S' \bar{g}_T' \rho} \geq \max \left\{ -\frac{u''_{TI}}{u'_{TI}}; -\frac{u''_{SI}}{u'_{SI}} \right\}$. Then, $\hat{g}_p^E$, g_S^E and g_T^E are complementary in λ_S , λ_T and $\hat{N}_C$. If it is also $\frac{1}{\rho \bar{g} \rho} \geq \max \left\{ -\frac{u''_{TI}}{u'_{TI}}; -\frac{u''_{SI}}{u'_{SI}} \right\}$ then $\hat{g}_p^E$, g_S^E and g_T^E are also complementary in ρ . $\hat{g}_p^E$, g_S^E and g_T^E are never complementary in N_T and N_S .

Proposition 2 shows that the co-movement of the traditional public good of social innovation depends on both of the property of the utility functions, and in particular their “curvatures”, and the property of the social innovation production function. Variations that reduce the contribution cost for the two groups involved in the production of social innovation (λ_S and λ_T), or increases the contribution productivity (ρ), unambiguously increases social innovation (and reduces the production of the traditional public good) only if the contribution of the two groups are complimentary *and* the marginal utility of social innovation is “large enough”. In other words, the complementarity of contributions is a necessary but not sufficient condition. It turns out that social innovation must also satisfy “unmet social needs”, so that an increase in social innovation may have a significant impact for the groups. As for the impact of group sizes, an increase in the size of the “normal” citizen group increases the production of the traditional public good (since the government cares about the utility obtained by this group), and reduces the production of social innovation. Variations in the size of the target group and of the social innovation group, instead, have an ambiguous effect. On the one hand, the government is induced to invest more in the traditional public good since it is enjoyed by a larger number of citizens; on the other hand, the target group and social innovators increase their contribution since the cost can be spread over more members.

3. Two special cases

In section 2.1.1, we specified how preferences over the traditional public good and social innovation for S and C members can be justified in terms of altruism in favor of the target group, and, for social innovators, by the nature of social innovation as a relational good. In this section, we further analyze two special case of the model presented in Section 2 that focuses in turn on each motivation.

In Section 3.1 we characterize the results for a model in which the traditional public good and social innovation are perfect substitute for the target group (so that only total amount of public good matters) and social innovators and normal citizens are positively influenced by the utility that the members of group T obtain. In Section 3.2, only social innovation (as a relational good) enters the utility function of S members, while the traditional public neither good nor social innovation enters the utility function of C members.

3.1 Social innovation and the role of altruism

In order to focus on the role of altruism for social motivation, we consider specific forms of utility functions as follows:

$$U_T(g, x_T) = u_T(g) + x_T \quad (16)$$

$$U_S(g, x_S) = \beta_S(N_T)u_T(g) + x_S \quad (17)$$

$$U_C(g, x_C) = \beta_C(N_T)u_T(g) + x_C \quad (18)$$

where $g = g_P + g_I \cdot \beta_S(N_T)$ and $\beta_C(N_T)$ (with $0 < \beta_S(N_T) < 1$ and $0 < \beta_C(N_T) < 1$, $\beta'_S(N_T) \geq 0$, $\beta'_C(N_T) \geq 0$, $\beta''_S(N_T) \leq 0$ and $\beta''_C(N_T) \leq 0$, are parameters measuring the degree of altruism towards the members of the target group. According to (16)-(18), members of group T are indifferent between the traditional public good and social innovation, and so are members of groups S and C .

We allow members of T and C to account non-linearly to for the size of target group. Given (16), (17), (18), the first order condition for social welfare maximization boils down to:

$$u_T'(g_P^* + g_I^*) = \frac{1}{N_T + \beta_C(N_T)N_C + \beta_S(N_T)N_S} \quad (19)$$

$$\rho \bar{G}'_S(g_S^*, g_T^*) u_T'(g_P^* + g_I^*) = \frac{1}{N_T + \beta_C(N_T)N_C + \beta_S(N_T)N_S} \quad (20)$$

$$\rho \bar{G}'_T(g_S^*, g_T^*) u_T'(g_P^* + g_I^*) = \frac{1}{N_T + \beta_C(N_T)N_C + \beta_S(N_T)N_S} \quad (21)$$

The total amount of public good is positively affected by the size of each group and the degree of altruism.

The first order conditions which determine the Nash equilibrium of the game are:

$$u_T'(g_P^E + g_{ST}^E) = \frac{1}{N_T + \beta_C(N_T)N_C + \beta_S(N_T)N_S} \quad (22)$$

$$\rho \bar{G}_S'(g_S^E, g_T^E) \beta_S(N_T) u_T'(g_P^E + g_I^E) = \frac{1 - \lambda_S}{N_S} \quad (23)$$

$$\rho \bar{G}_T'(g_S^E, g_T^E) u_T'(g_P^E + g_I^E) = \frac{1 - \lambda_T}{N_T} \quad (24)$$

We observe that (19) and (22) coincide, which implies the total level of the public good in the Nash equilibrium is at the social optimal level. However, from Proposition 1, the *composition* of the public good, in terms of traditional public good and social innovation, will differ between the Nash equilibrium and the social optimum.

3.1.1 Comparative statics

In this section, we derive standard comparative statics exercises with respect model parameters.

First, we observe that plugging (22) into (23) and (24) we obtain:

$$\rho \bar{G}_S'(g_S^E, g_T^E) = [N_T + \beta_C(N_T)N_C + \beta_S(N_T)N_S] \frac{1 - \lambda_S}{\beta_S(N_T)N_S} \quad (25)$$

$$\rho \bar{G}_T'(g_S^E, g_T^E) = [N_T + \beta_C(N_T)N_C + \beta_S(N_T)N_S] \frac{1 - \lambda_T}{N_T} \quad (26)$$

By applying the implicit function theorem on the system (25)-(26) we will instigate the impact on S and T members' contribution g_S and g_T to social innovation by:

- The level of subsidies, λ_S and λ_T
- The productivity parameter ρ
- The total altruistic utility enjoyed by group S and C (per member of T), defined as $\tilde{\beta}_S = \beta_S(N_T)N_S$ and $\tilde{\beta}_C = \beta_S(N_T)N_C$.
- The size of the target group N_T .

The results are summarized by the following propositions:

Proposition 3 i) An increase in λ_S (λ_T) has always a positive impact on g_S^E (g_T^E), and a positive impact on g_T^E , (g_S^E) if $G_{ST}'' > 0$ (negative otherwise).

- ii) An increase in ρ has a positive impact on g_S^E (g_T) if $G_{ST}'' > 0$, and an ambiguous sign otherwise.
- iii) An increase in $\tilde{\beta}_S$ has a positive (negative) impact on g_S^E (g_T^E) if $G_{ST}'' < 0$, and an ambiguous impact otherwise; an increase in $\tilde{\beta}_C$ has a negative impact on g_S^E (g_T^E) if $G_{ST}'' > 0$, and an ambiguous impact otherwise.
- iv) An increase in N_T has in general an ambiguous effect on g_S^E (g_T^E). The impact is positive if $G_{ST}'' < 0$ and $\beta'_S(N_T) = 0$ and $\beta'_C(N_T) = 0$ and nihil if $\beta''_S(N_T) = 0$ and $\beta''_C(N_T) = 0$.

Proposition 3 show how comparative statics crucially depend on the properties of social innovation production function.

Point i) in Proposition 3 shows, unsurprisingly, that subsidies stimulate the effort of the group who receive it, but it has the same effect on other group when efforts are complimentary.

Point ii) of Proposition 3 shows that an increase in the total productivity of S and T contributions to social innovation has the effect of increasing efforts only if efforts are complimentary, as contribution by members in S and T would reinforce each other in this case. If they are substitute, the more productive contribution (in terms of social innovation production) would increase, while the other would decrease.

Point iii) in the Proposition 3 shows the impact of an increase in the total altruistic utility obtained by S members. For S members, an increase of $\tilde{\beta}_S$ generates two opposing forces: i) a direct (positive) effect of increasing the contribution by S members ii) an increase in the desired total level of the public good by the local government, which crowds out the investment in social innovation. It turns out the first effect is stronger. For T members, only the latter is present. It follows that when contribution are substitutes, contribution by T members would decrease, leading the contribution by T members to increase unambiguously. When contributions are complimentary, the net effect is indeterminate. As for the impact of $\tilde{\beta}_C$, the direct effect is to increase the desired total level of the public good by the local government, inducing a reduction in social innovation contributions. When the two contribution are complimentary, the effects on S and T reinforce each other. In this case, social innovation would be play a significant role when the target group needs are not perceived as relevant by the community in general. When contributions are substitute, one contribution could indeed compensate for the other.

Finally, as for point iv) the impact of target group size has an ambiguous effect on social innovation since it increases the marginal benefit of social innovators and reduces the marginal cost for the

investment by T members, but it also increases the traditional public good level. Ambiguity is solved in two cases. First, when total altruistic utility is not affected by the size of the target group, for the target group the reduction in the contribution cost prevails on the increased incentive of the government to invest in the traditional public good. For the social innovators, the direct impact is negative since this last effect is only one that is present. When contributions are substitutes, then, T members contribute more and S members less.

Second, if total altruistic is linearly increasing in the target group size, then social innovation level is unaffected, as the various effect compensate each other. However, in that case, as the total level of public good increases with N_T , it follows social innovation would be *relatively* less important as the size of the target group increases.

3.1.2 Optimal social innovation policy

The optimal policy of the local government consists in pair of subsidies $(\lambda_T^*; \lambda_S^*)$, chosen to maximize social welfare before the policy game is played and correctly predicting the impact on the Nash equilibrium.

Proposition 4 *Optimal subsidies are given by $\lambda_S^* = \frac{N_T + \beta_C(N_T)N_C}{N_T + \beta_S(N_T)N_S + \beta_C(N_T)N_C}$ and $\lambda_T^* = \frac{\beta_S(N_T)N_S + \beta_C(N_T)N_C}{N_T + \beta_S(N_T)N_S + \beta_C(N_T)N_C}$*

As we knew already from Proposition 1, optimal subsidies are always non-negative. Proposition 4 shows how they depend on the parameters for this specific model. First we observe that the optimal subsidy levels do not depend on the properties of the social innovation production, but only on the size and the degree of altruism of the various groups. Moreover, $\lambda_S^* + \lambda_T^* = 1$.

In addition, it is straightforward to show $\frac{d\lambda_S^*}{d\beta_S} < 0$ and $\frac{d\lambda_T^*}{d\beta_S} > 0$. When the total altruistic utility for S increases, the optimal subsidy for the S group decreases because of the increase in its intrinsic motivation, while the subsidy for the T group increases because it is socially desirable to increase the total investment in the public good, and the government must align social and private incentives of T group investment.

Finally, we can show $\frac{d\lambda_S^*}{d\beta_C} > 0$ and $\frac{d\lambda_T^*}{d\beta_C} > 0$. When the total spillovers for C increases, the optimal subsidy for the S and T groups decreases because it is socially desirable to increase the investment

in the public good, and the government must align social and private incentives for both T and S group investments.

3.2 Social innovation as a relational good

In this section, we fully characterize a special model in which social innovators are exclusively motivated by the “consumption” of social innovation as a relational good (Ulhaner, 1989). Preferences are thus represented by the following utility functions:

$$U_T(g_P, g_I, x_T) = u_I(g_I) + u_T(g_P) + x_T \quad (25)$$

$$U_S(g_P, g_I, x_S) = u_I(g_I) + x_S \quad (26)$$

$$U_C(g, x_C) = x_C \quad (27)$$

Given (25), (26) and (27), the first order condition for social welfare maximization boils down to:

$$N_T u'_T(g_P^*) = 1 \quad (28)$$

$$N_S u'_I(g_I^*) \rho \bar{G}'_S(g_S^*, g_T^*) = 1 \quad (29)$$

$$N_T u'_I(g_I^*) \rho \bar{G}'_S(g_S^*, g_T^*) = 1 \quad (30)$$

As for the condition for the Nash equilibrium, we obtain:

$$N_T u'_T(g_P^E) = 1 \quad (31)$$

$$N_S u'_I(g_I^*) \rho \bar{G}'_S(g_S^E, g_T^E) = 1 - \lambda_S \quad (32)$$

$$N_T u'_I(g_I^*) \rho \bar{G}'_S(g_S^E, g_T^E) = 1 - \lambda_T \quad (33)$$

3.2.1 Comparative statics

By applying the implicit function theorem on the system (31)-(32) we will instigate the impact on S and T members' contribution to social innovation g_S and g_T by:

- The level of subsidies, λ_S and λ_T .
- The productivity parameter ρ .
- The group sizes of N_S and N_T .

The results are summarized by the following propositions:

Proposition 5 i) An increase in λ_S (λ_T) has always a positive impact on g_S^E (g_T^E), and a positive impact on g_T^E (g_S^E) if $G''_{ST} > 0$ (negative otherwise).

ii) An increase in ρ has a positive impact on g_S^E (g_T^E) if $G''_{ST} > 0$, and an ambiguous sign otherwise.

iii) An increase in N_S (N_T) has always a positive impact on g_S^E (g_T^E), and a positive impact on g_T^E (g_S^E) if $G_{ST}'' > 0$ (negative otherwise).

3.2.2 Optimal social innovation policy

By comparing (28)-(30) to (31)-(33) Proposition 6 easily derives.

Proposition 6 *Optimal subsidies are given by $\lambda_S^* = 0$ and $\lambda_T^* = 0$*

When social innovation takes the form of a relational good, in fact there are not positive externalities to internalize.

4. Discussion

4.1 Social innovation when traditional public goods do not exist

The assumption the government maximizes social welfare may be criticized from a political economy perspective. For instance, the government could maximize the welfare the median voter.

If we assume $\frac{N_C}{N_C + N_T + N_S} > \frac{1}{2}$ and $U_C(g_P, g_P) = 0$, then $g_P^* = 0$ (and $\lambda_S^* = \lambda_T^* = 0$). If we compare the level of social innovation resulting from the optimal social innovation policy with that resulting from no investment in the traditional public good, which one be higher?

It turns out that the answer is ambiguous. If we plug $g_P^* = 0$ and $\lambda_S^* = \lambda_T^* = 0$ into (14) and (15) we obtain:

$$N_S u'_{SI}(0, g_I^E) G'_S(g_S^E, g_T^E) = 1 \quad (34)$$

$$N_T u'_{TI}(0, g_I^E) G'_T(g_S^E, g_T^E) = 1 \quad (35)$$

We observe that both the marginal benefit and the marginal cost of contributions to social innovation are higher (since u''_{SPI} and u''_{TPI} are negative

In the specific model where social motivation is motivated by altruism, the results can be more characterized. Absent a traditional public good, the conditions for Nash equilibrium become:

$$\rho \beta_S(N_T) u'_T(\rho g_I^*) G'_S = \frac{1}{N_S} \quad (36)$$

$$\rho u'_T(g_I^*) G'_T = \frac{1}{N_T} \quad (37)$$

with $u_T'(g_{ST}^*) < 1$ (since $u_T'(g_P^{FB} + g_I^{FB}) = \frac{1}{N_T + \beta_C(N_T)N_C + \beta_S(N_T)N_T} < 1$). If we compare the first order conditions for S and T with optimal subsidy (for which $\rho G_S' = \rho G_T' = 1$) we obtain that T group always invests more when the government does not offer g_P^* , while the S group invests more when $\beta_S N_T u_T'(\rho g_{ST}^*) > 1$.

4.2 Can social innovation reduce taxation?

The political support for social innovation requires to look not only at total welfare, but also at the impact of social innovation for each group.

Suppose we consider the special model analyzed in Section 4.1, in which social innovation is motivated by altruism, and assume that $\beta_C = 0$. For common citizens, then, the only impact of social innovation is the through the impact on taxation, which members of group C pay for producing the traditional public good and to pay subsidies.

If we compare a situation in which social innovation does not exist (say because $\rho = 0$) to the case in which it does, and it is optimally subsidized (when $\rho > 0$), is it possible that taxes paid by C members are actually lower?

The answer is affirmative. Let us define $\bar{g}$ as the socially optimal total level of public good. From equation (9), absent social innovation, we obtain

$$\tau = \frac{\bar{g}}{(N_S + N_C)} \quad (38)$$

Instead, if social innovation is optimally subsidized we obtain:

$$\tau = \frac{\bar{g} - g_I^* + \lambda_S^* g_S^* + \lambda_T^* g_T^*}{(N_S + N_C)} \quad (39)$$

Easy comparison shows taxation is reduced by introducing social innovation (with optimal social innovation policy) when $g_I^* - \lambda_S^* g_S^* - \lambda_T^* g_T^* > 0$, i.e. when the contribution to social innovation are sufficiently productive. For instance, suppose that $g_I = \rho g_S g_T$. In this case it is $g_S^* = g_T^* = \rho$. It follows that $g_I^* - \lambda_S^* g_S^* - \lambda_T^* g_T^* = \rho^3 - \rho > 0$ when $\rho > 1$.

5. Conclusions

In this paper, we provide a simple theoretical model to analyse social innovation as the *community provision* of public goods, presenting a formalization which is compatible with the main insights and results of the literature on social innovation in social sciences other than economics. We mainly

focus on public policy towards social innovation, but also consider possible testable implications. From a welfare point of view, traditional public good and social innovation should coexist, with governments also involved in actively supporting social innovation via subsidies. We also show that each community member can be better off with social innovation, as relying on social innovators' intrinsic motivations can lower taxes used to finance traditional public goods and social innovation subsidies. Finally, we show that the optimal policy towards social innovation typically depends on the social innovation co-production function characteristics.

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Proof of Proposition 1

By evaluating $\frac{dW}{dg_P}, \frac{dW}{dg_S}, \frac{dW}{dg_T}$ at g_P^E, g_S^E, g_T^E we obtain:

$$\frac{dW}{dg_P} \equiv N_T u'_{TP}(g_P^E, g_I^E) + N_S u'_{SP}(g_P^E, g_I^E) + N_C u'_{CP}(g_P^E, g_I^E) = 1 \quad (A1)$$

$$\frac{dW}{dg_S} \equiv N_T u'_{TI}(g_P^E, g_I^E) G'_S(g_S^E, g_T^E) + N_C u'_{CI}(g_P^E, g_I^E) G'_S(g_S^E, g_T^E) \geq 0 \quad (A2)$$

$$\frac{dW}{dg_T} \equiv N_S u'_{SI}(g_P^E, g_I^E) G'_T(g_S^E, g_T^E) + N_C u'_{CI}(g_P^E, g_I^E) G'_T(g_S^E, g_T^E) \geq 0 \quad (A3)$$

It follows that $g_S^E \leq g_T^*$ and $g_T^E \leq g_T^*$. This implies $g_I^E < g_I^*$. Since $u''_{TPI} \leq 0$, $u''_{TPI} \leq 0$ and $u''_{TPI} \leq 0$ then $g_P^E > g_P^*$.

Proof of Proposition 2

The proposition follows from the inspection of cross derivatives:

$$\frac{\partial^2 W}{\partial \hat{g}_P \partial g_S} \equiv -N_T [u''_{TPI} \rho \bar{G}'_S] - N_S [u''_{TPI} \rho \bar{G}'_S] - N_C [u''_{TPI} \rho \bar{G}'_S] > 0 \quad (A4)$$

$$\frac{\partial^2 W}{\partial \hat{g}_P \partial g_T} \equiv -N_T [u''_{TPI} \rho \bar{G}'_T] - N_S [u''_{TPI} \rho \bar{G}'_T] - N_C [u''_{TPI} \rho \bar{G}'_T] > 0 \quad (A5)$$

$$\frac{\partial^2 W}{\partial \hat{g}_P \partial \rho} \equiv -N_T [u''_{TPI} \bar{G}_T] - N_S [u''_{TPI} \bar{G}_T] - N_C [u''_{TPI} \bar{G}_T] > 0 \quad (A6)$$

$$\frac{\partial^2 W}{\partial \hat{g}_P \partial \lambda_S} = \frac{\partial^2 W}{\partial \hat{g}_P \partial \lambda_T} = 0 \quad (A7)$$

$$\frac{\partial^2 U_T}{\partial g_T \partial \hat{g}_P} \equiv -N_T [u''_{TPI} \rho \bar{G}'_T] > 0 \quad (A8)$$

$$\frac{\partial^2 U_S}{\partial g_S \partial \hat{g}_P} \equiv -N_S [u''_{SPI} \rho \bar{G}'_S] > 0 \quad (A9)$$

$$\frac{\partial^2 U_T}{\partial g_T \partial \lambda_T} = \frac{\partial^2 U_S}{\partial g_S \partial \lambda_S} = 1 \quad (A10)$$

$$\frac{\partial^2 U_T}{\partial g_T \partial \lambda_T} = \frac{\partial^2 U_S}{\partial g_S \partial \lambda_S} = 0 \quad (A11)$$

$$\frac{\partial^2 W}{\partial \hat{g}_P \partial N_C} = -u'_{CP} < 0 \quad (A12)$$

$$\frac{\partial^2 W}{\partial \hat{g}_P \partial N_S} = -u'_{SP} < 0 \quad (A13)$$

$$\frac{\partial^2 W}{\partial \hat{g}_P \partial N_T} = -u'_{TP} < 0 \quad (A14)$$

$$\frac{\partial^2 U_T}{\partial g_T \partial g_S} \equiv \rho N_T [u''_{TI} \rho \bar{G}'_T \bar{G}'_T + u'_{TI} \bar{G}''_{ST}] \quad (A15)$$

$$\frac{\partial^2 U_S}{\partial g_T \partial g_S} \equiv \rho N_S [u''_{SI} \rho \bar{G}'_S \bar{G}'_S + u'_{SI} \bar{G}''_{ST}] \quad (A16)$$

$$\frac{\partial^2 U_T}{\partial g_T \partial \rho} \equiv \bar{G}'_T N_T [u''_{TI} \rho \bar{G} + u'_{TI}] \quad (\text{A17})$$

$$\frac{\partial^2 U_S}{\partial g_S \partial \rho} \equiv \bar{G}'_S N_S [u''_{SI} \rho \bar{G} + u'_{SI}] \quad (\text{A18})$$

The conditions specified in the paper guarantees that (A15)-(A18) are all positive.

Proof of Proposition 3

(25) and (26) can be re-written as the two-equation systems:

$$\rho \tilde{\beta}_S \frac{\bar{G}'_S}{(N_T + \tilde{\beta}_S + \tilde{\beta}_C)} - (1 - \lambda_S) = 0 \quad (\text{A19})$$

$$\rho \frac{\bar{G}'_T}{(N_T + \tilde{\beta}_S + \tilde{\beta}_C)} - (1 - \lambda_T) = 0 \quad (\text{A20})$$

i) By applying the implicit function theorem on this system, we obtain:

$$\frac{dg_S^E}{d\lambda_S} = - \frac{(N_T + \tilde{\beta}_S + \tilde{\beta}_C) \bar{G}''_T}{\rho \tilde{\beta}_S [\bar{G}''_S \bar{G}''_T - (\bar{G}''_{ST})^2]} > 0 \quad (\text{A21})$$

since $\bar{G}''_S \bar{G}''_T - (\bar{G}''_{ST})^2 > 0$ for the stability of the Nash equilibrium, and

$$\frac{dg_T^E}{d\lambda_S} = \frac{(N_T + \tilde{\beta}_S + \tilde{\beta}_C) \bar{G}''_{ST}}{\rho \tilde{\beta}_S [\bar{G}''_S \bar{G}''_T - (\bar{G}''_{ST})^2]} \quad (\text{A22})$$

which has the same sign of $\bar{G}''_{ST} > 0$.

Similarly,

$$\frac{dg_S^E}{d\lambda_T} = \frac{(N_T + \tilde{\beta}_S + \tilde{\beta}_C) \bar{G}''_{ST}}{\rho \tilde{\beta}_S [\bar{G}''_S \bar{G}''_T - (\bar{G}''_{ST})^2]} \quad (\text{A23})$$

has the same sign of $\bar{G}''_{ST}$, while:

$$\frac{dg_T^E}{d\lambda_T} = - \frac{(N_T + \tilde{\beta}_S + \tilde{\beta}_C) \bar{G}''_S}{\rho \tilde{\beta}_S [\bar{G}''_S \bar{G}''_T - (\bar{G}''_{ST})^2]} > 0 \quad (\text{A24})$$

ii) By applying the implicit function theorem we obtain:

$$\frac{dg_S^E}{d\rho} = \frac{\bar{G}_{ST}'' \bar{G}_T' - \bar{G}_S' \bar{G}_T''}{(N_T + \tilde{\beta}_S + \tilde{\beta}_C) [\bar{G}_S'' \bar{G}_T' - (\bar{G}_{ST}'')^2]} \quad (A25)$$

$$\frac{dg_T^E}{d\rho} = \frac{\bar{G}_{ST}'' \bar{G}_S' - \bar{G}_T' \bar{G}_S''}{(N_T + \tilde{\beta}_S + \tilde{\beta}_C) [\bar{G}_S'' \bar{G}_T' - (\bar{G}_{ST}'')^2]} \quad (A26)$$

which are positive if $\bar{G}_{ST}'' > 0$, while the sign is indeterminate if $\bar{G}_{ST}'' < 0$.

iii) By applying the implicit function theorem we obtain:

$$\frac{dg_S^E}{d\tilde{\beta}_S} = \frac{-\bar{G}_{ST}'' \bar{G}_T' - \bar{G}_S' \bar{G}_T''}{(1 + \tilde{\beta}_S + \tilde{\beta}_C) [\bar{G}_S'' \bar{G}_T' - (\bar{G}_{ST}'')^2]} \quad (A9)$$

$$\frac{dg_T^E}{d\tilde{\beta}_S} = \frac{\bar{G}_{ST}'' \bar{G}_S' - \bar{G}_T' \bar{G}_S''}{(1 + \tilde{\beta}_S + \tilde{\beta}_C) [\bar{G}_S'' \bar{G}_T' - (\bar{G}_{ST}'')^2]} \quad (A10)$$

which are positive if $\bar{G}_{ST}'' < 0$ (while the sign is indeterminate otherwise).

Similarly,

$$\frac{dg_S^E}{d\tilde{\beta}_C} = \frac{-\bar{G}_{ST}'' \bar{G}_T' + \bar{G}_S' \bar{G}_T''}{(1 + \tilde{\beta}_S + \tilde{\beta}_C) [\bar{G}_S'' \bar{G}_T' - (\bar{G}_{ST}'')^2]} \quad (A11)$$

$$\frac{dg_T^E}{d\tilde{\beta}_C} = \frac{-\bar{G}_{ST}'' \bar{G}_S' + \bar{G}_T' \bar{G}_S''}{(1 + \tilde{\beta}_S + \tilde{\beta}_C) [\bar{G}_S'' \bar{G}_T' - (\bar{G}_{ST}'')^2]} \quad (A12)$$

which are negative if $\bar{G}_{ST}'' > 0$ (while the sign is indeterminate otherwise).

iv) [TO BE WRITTEN]

Proof of Proposition 4

λ_S^* and λ_T^* are easily obtained plugging $\rho \bar{G}_T'(g_S^E, g_T^E) = 1$ and $\rho \bar{G}_S'(g_S^E, g_T^E) = 1$ into the system (25)-(26) and solving.

Proof of Proposition 5

[TO BE WRITTEN]

Proof of Proposition 6

The result is immediate by comparing (29) and (30) to (32) and (33).

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